

Evaluation of Compound Options in an Ambiguous

GeethikaN

India

Abstract: The authors Wang, He, and Li created a method for pricing compound options in a fuzzy setting, and they published their findings in the Journal of Applied Mathematics (Volume 2014, Article ID 875319, 9 pages). On the other hand, we discovered that they unconsciously thought that fuzzy addition and fuzzy division commutated with the crisp probabilistic mean value. We demonstrate in this study that their assumptions are flawed, exposing the need for additional development in their theoretical derivations.

Keywords: Complex Option Pricing, Uncertainty

Introduction

Numerous writers have attempted to generalize Geske's (1979) closed-form pricing formula for compound options to new contexts. As an example, numerous studies have examined the valuation of compound options. These include works by Geske and Johnson (1984), Thomassen and Wouwe (2001), Lajeri-Chaherli (2002), Lin (2002), Cassimon et al. (2004), Gukhal (2004), Agliardi and Agliardi (2005), Fouque and Han (2005), Lee et al. (2008), Chiarella and Kang (2011), Griebisch (2013), and Park et al. (2013). Sequential compound option techniques were examined by Carr (1988), Paxson (2007), and Huang and Pi (2009). For their 2003 work, Agliardi & Agliardi took Geske's model into account, as did Chen.

1. Review of previous results

Based on the pricing formula for compound option of Geske (1979), Wu (2004) and Nowak and Romaniuk (2010), Wang et al. (2014) developed their compound option pricing under fuzzy environment. We directly cite their results in the next theorem. To save the precious space of the journal, we only list those related results. For a detailed derivation of Wang et al. (2014), please refer to the original paper of Wang et al. (2014). **Theorem 4 of Wang et al. (2014)**

Let the interest rate and the volatility be fuzzy numbers. Then the fuzzy price of compound option Yoshida (2003), Yoshida et al. (2006), Chrysafis and Our discussion for the sensitivity analysis of Wang et al. (2014)

At last, for completeness, we will point out another questionable result in the sensitivity analysis of Wang et al. (2014). They compared (a) the underlying asset price, S_* and

the compound option price, C under the Black-Scholes model in their Table 1, and (b) the corresponding S^* and (\sim) of their model under fuzzy environment in their Table 2. $M C$ We observe their Tables 1 and 2 to know that $S \geq S^*$ and $C \leq M(\tilde{C})$. Our observation was contradicted with assertion mentioned in Wang et al. (2014).

For completeness, we quote their results, except the index number of referred two articles are modified to be consistent within this paper, "From Tables 1 and 2, the compound option prices derived from the Black-Scholes model are slightly lower than the prices derived from the crisp possibilistic mean value with the same parameters. This seems to be consistent with our intuition that the crisp possibilistic mean value model contains more uncertainty than the Black-Scholes model (see

Xu et al. (2009), Zhang et al. (2012)). But this intuition is not necessarily true, which one is bigger between C and (\sim) is related to the selected parameters. Similarly, from Tables 1 and 2, we notice that S_* is slightly higher than S ; this conclusion is not surely true. For example, when $S = 100$,

$K_1 = 5$, $K_2 = 90$, $T_1 = 0.5$, $T_2 = 1$, $\tilde{r} = (0.049, 0.05, 0.052)$

and $\sigma \sim = (0.28, 0.3, 0.31)$, then the computing result is Sets and

$M(C) = 15.2290$; obviously, $S < S^*$ and $C > M(C)$."

From our partially cited Tables 1 and 2 of Wang et al. (2014) as our table 1, we find that $S_* = 82.8336$ and $C = 15.2744$ as reported in the above citation.

However, on the contrary, in their Table 2 (cited in our table 1), $S^* = 82.7509$ and $(\sim) = 15.3199$ such that their claim of $S < S^*$ and $C > (\sim)$ contains $M C$ questionable findings.

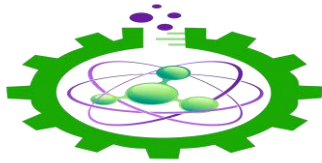


Table 1. Partially cited results from Tables 1 and 2 of Wang et al. (2014)

Table 1			Table 2		$M(\tilde{C})$
T_2	S_*	C	T_2	S^*	
0.75	88.2795	13.6596	0.75	88.2253	13.7005
1	82.8336	15.2744	1	82.7509	15.3199
1.25	78.3652	16.8882	1.25	78.2638	16.9375

$S = 100$, $K_1 = 5$, $K_2 = 90$, $T_1 = 0.5$, $r = 0.05$, $\sigma = 0.3$,
 $r_c = 0.05$, and $\sigma_c = 0.3$.

2. Conclusions

Based on our above discussion, we prove that

$M(a \otimes \tilde{b}) = M(a)M(\tilde{b})$ if and only if \tilde{a} or \tilde{b} are crisp numbers.

Moreover, for triangular fuzzy numbers, in general,

$M(a \otimes \tilde{b}) \neq M(\tilde{a})M(\tilde{b})$. On the other hand, based on equations (34) and (35), $M(a \div \tilde{b})$ and $M(a \tilde{b})$ are completely

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